

# Combining red and blue-detuned optical potentials to form a Lamb-Dicke trap for a single neutral atom

Xiaodong He,<sup>1,2</sup> Shi Yu,<sup>1,2,3</sup> Peng Xu,<sup>1,2</sup> Jin Wang,<sup>1,2</sup>  
and Mingsheng Zhan<sup>1,2,\*</sup>

<sup>1</sup>State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences - Wuhan National Laboratory for Optoelectronics, Wuhan 430071, China

<sup>2</sup>Center for Cold Atom Physics, Chinese Academy of Sciences, Wuhan 430071, China

<sup>3</sup>Graduate University of the Chinese Academy of Sciences, Beijing 100049, China

\*[mszhan@wipm.ac.cn](mailto:mszhan@wipm.ac.cn)

**Abstract:** We propose and demonstrate a scheme for strong radial confinement of a single  $^{87}\text{Rb}$  atom by a bichromatic far-off resonance optical dipole trap (BFORT). The BFORT is composed of a blue-detuned Laguerre-Gaussian  $LG_0^1$  beam and a red-detuned Gaussian beam. The atomic oscillation frequency measurement shows that the effective trapping dimension is much sharper than that from a diffraction-limited microscopic objective. Theory shows that the added scattering rate due to imposing blue-detuned light is negligible when the temperature of the single atoms is close to ground state temperature. By carrying out sub-Doppler cooling, the mean energy of single atoms trapped in the BFORT is reduced to  $15 \pm 1 \mu\text{K}$ . The corresponding mean quantum number of radial vibration  $\bar{n}$  is about 1.65, which satisfies the Lamb-Dicke regime. We conclude that the BFORT is a suitable Lamb-Dicke trap for further cooling a single neutral atom down to the ground state and for further application in quantum information processing.

© 2012 Optical Society of America

**OCIS codes:** (020.3320) Laser cooling; (020.7010) Laser trapping; (270.5585) Quantum information and processing; (090.2890) Holographic optical elements.

---

## References and links

1. D. Frese, B. Ueberholz, S. Kuhr, W. Alt, D. Schrader, V. Gomer, and D. Meschede, "Single atoms in an optical dipole trap: Towards a deterministic source of cold atoms," *Phys. Rev. Lett.* **85**, 3777–3780 (2000).
2. N. Schlosser, G. Reymond, I. Protsenko, and P. Grangier, "Sub-poissonian loading of single atoms in a microscopic dipole trap," *Nature (London)* **411**, 1024–1027 (2001).
3. D. Schrader, I. Dotsenko, M. Khudaverdyan, Y. Miroshnychenko, A. Rauschenbeutel, and D. Meschede, "Neutral atom quantum register," *Phys. Rev. Lett.* **93**, 150501 (2004).
4. Y. Miroshnychenko, W. Alt, I. Dotsenko, L. Forster, M. Khudaverdyan, D. Meschede, D. Schrader, and A. Rauschenbeutel, "Quantum engineering: An atom-sorting machine," *Nature (London)* **442**, 151 (2006).
5. J. Beugnon, C. Tuchendler, H. Marion, A. Gaetan, Y. Miroshnychenko, Y. R. P. Sortais, A. M. Lance, M. P. A. Jones, G. Messin, A. Browaeys, and P. Grangier, "Two-dimensional transport and transfer of a single atomic qubit in optical tweezers," *Nat. Phys.* **3**, 696–699 (2007).
6. D. Leibfried, R. Blatt, C. Monroe, and D. J. Wineland, "Quantum dynamics of single trapped ions," *Rev. Mod. Phys.* **75**, 281–324 (2003).
7. D. Jaksch, H.-J. Briegel, J. I. Cirac, C. W. Gardiner, and P. Zoller, "Entanglement of atoms via cold controlled collisions," *Phys. Rev. Lett.* **82**, 1975–1978 (1999).

8. Y. R. P. Sortais, H. Marion, C. Tuchendler, A. M. Lance, M. Lamare, P. Fournet, C. Armellin, R. Mercier, G. Messin, A. Browaeys, and P. Grangier, "Diffraction-limited optics for single-atom manipulation," *Phys. Rev. A* **75**, 013406 (2007).
9. C. Tuchendler, A. M. Lance, A. Browaeys, Y. R. P. Sortais, and P. Grangier, "Energy distribution and cooling of a single atom in an optical tweezer," *Phys. Rev. A* **78**, 033425 (2008).
10. T. A. Savard, K. M. OHara, and J. E. Thomas, "Laser-noise-induced heating in far-off resonance optical traps," *Phys. Rev. A* **56**, R1095–R1098 (1997).
11. S. J. M. Kuppens, K. L. Corwin, K. W. Miller, T. E. Chupp, and C. E. Wieman, "Loading an optical dipole trap," *Phys. Rev. A* **62**, 013406 (2000).
12. N. Friedman, A. Kaplan, and N. Davidson, "Dark optical traps for cold atoms," *Adv. At. Mol. Opt. Phys.* **48**, 99–151 (2002).
13. X. D. He, P. Xu, J. Wang, and M. S. Zhan, "Rotating single atoms in a ring lattice generated by a spatial light modulator," *Opt. Express* **17**, 21007–21014 (2009).
14. M. Weber, J. Volz, and K. Saucke, "Analysis of a single-atom dipole trap," *Phys. Rev. A* **73**, 043406 (2006).
15. P. Xu, X. D. He, J. Wang, and M. S. Zhan, "Trapping a single atom in a blue detuned optical bottle beam trap," *Opt. Lett.* **35**, 2164–2166 (2010).
16. L. D. Landau and E. M. Lifshitz, *Mechanics*, 3rd ed. (Butterworth-Heinemann, 1998).
17. R. Grimm, M. Weidemuller, and Y. B. Ovchinnikov, "Optical dipole traps for neutral atoms," *Adv. At. Mol. Opt. Phys.* **42**, 95–170 (2000).
18. R. A. Cline, J. D. Miller, M. R. Matthews, and D. J. Heinzen, "Spin relaxation of optically trapped atoms by light scattering," *Opt. Lett.* **19**, 207–209 (1994).
19. C. Cabrillo, J. I. Cirac, P. Garcia-Fernandez, and P. Zoller, "Creation of entangled states of distant atoms by interference," *Phys. Rev. A* **59**, 1025–1033 (1999).
20. C. Monroe, D. M. Meekhof, B. E. King, S. R. Jefferts, W. M. Itano, D. J. Wineland, and P. Gould, "Resolved-sideband raman cooling of a bound atom to the 3D zero-point energy," *Phys. Rev. Lett.* **75**, 4011–4014 (1995).
21. C. F. Roos, D. Leibfried, A. Mundt, F. Schmidt-Kaler, J. Eschner, and R. Blatt, "Experimental demonstration of ground state laser cooling with electromagnetically induced transparency," *Phys. Rev. Lett.* **85**, 5547–5550 (2000).
22. G. K. Brennen, C. M. Caves, P. S. Jessen, and I. H. Deutsch, "Quantum logic gates in optical lattices," *Phys. Rev. Lett.* **82**, 1060–1063 (1999).
23. X. D. He, P. Xu, J. Wang, and M. S. Zhan, "High efficient loading of two atoms into a microscopic optical trap by dynamically reshaping the trap with a spatial light modulator," *Opt. Express* **18**, 13586–13592 (2010).
24. L. Föster, M. Karski, J. M. Choi, A. Steffen, W. Alt, D. Meschede, and A. Widera, "Microwave control of atomic motion in optical lattices," *Phys. Rev. Lett.* **103**, 233001 (2009).

## 1. Introduction

Laser cooled neutral atoms in shallow red-detuned far-off resonance optical dipole traps (FORT) have long hyperfine state life times because of the weak interaction with the electromagnetic field [1]. The ability to trap single atoms in red-detuned optical tweezers and then manipulate the atomic external and internal degrees of freedom has opened the way to controlled engineering of the quantum state of neutral atoms [2–5]. These developments have made the neutral atoms one of the most promising candidates for storing and processing quantum information. As in the case of ions [6], we need to cool trapped single atoms to the ground vibrational state of the trapping potential in order to precisely control the internal and external degrees of freedom of the atoms. In quantum computing, the theoretically proposed gate operation using controlled collisions between neutral atoms usually requires ground state cooling [7].

All successful experiments on ground state cooling have featured an initial laser cooling stage that reached the Lamb-Dicke regime [6]. The Lamb-Dicke regime requires  $\eta^2(2n + 1) \ll 1$ , where  $\eta$  is Lamb-Dicke parameter and  $n$  is motional quantum number of the atomic harmonic oscillator state. The Lamb-Dicke parameter is given by  $\eta = \sqrt{\frac{E_r}{\hbar\omega}} \ll 1$ , where  $E_r$  is the recoil energy of atoms, and  $\omega$  is the angular oscillation frequency of trapped single atoms;  $\omega$  is related to the trap potential depth  $U$  and the waist of a focused laser beam  $w_0$  by  $\omega \propto \sqrt{U}/w_0$ . To construct a Lamb-Dicke trap for single neutral atoms and further laser-cool the atoms to the ground state, we can increase the oscillation frequency by raising the trap potential depth or reducing the waists of the focused spots. However a higher potential depth causes a larger

optical ac Stark shift which suppresses the fluorescence emission rate. So the trapping potential depths are typically set to less than 1 mK. Since the radial oscillation frequency depends on the waist more strongly than trap depth, seeking a sharper focus is a good approach to making a Lamb-Dicke trap for a single atom. However, for technical reasons, it is difficult to make the focus small enough. At present, by using diffraction-limited optics based on the combination of a large numerical aperture (NA=0.5) aspheric lens placed inside the vacuum chamber and a few standard lenses placed outside, a waist of  $w_0 = 1.03 \pm 0.01 \mu\text{m}@850 \text{ nm}$  and a radial oscillation frequency  $\omega_r/2\pi = 160 \pm 3 \text{ kHz}$  with a trap depth  $U_0 = 2.8 \text{ mK}$  were obtained [8,9]. These two experiments were carried out with single  $^{87}\text{Rb}$  atoms.

The results just described are close to the diffraction limit of the optical system. If stronger confinement is required and at the same time a lower red-detuned potential depth is desired then a different method must be used.

Here, we discuss a new approach to constructing a radially steep optical dipole trap for single  $^{87}\text{Rb}$  atoms by a non diffraction-limited optical system with NA=0.3 placed outside the vacuum chamber. The key to our scheme is the use of a bichromatic laser field with a combination of different transverse modes. Specially, we overlap a blue-detuned Laguerre-Gaussian laser beam ( $LG_0^1$ ) with a red-detuned Gaussian beam. The atoms experience a repulsive force from the blue-detuned doughnut beam and an attractive force from the red-detuned Gaussian beam; both forces push the atoms toward the optical axis. Together, the two beams create an ultra steep potential for single neutral atoms. We call this complex trap a bichromatic far-off resonance optical dipole trap (BFORT). We realized a BFORT and measured the oscillation frequencies by parametric excitation [10]. With increased potential depth provided by the blue-detuned doughnut beam, a single atom oscillation frequency that was 50 kHz in a simple FORT can be enhanced by a factor of 3. The corresponding effective waist  $w_{eff}$  was compressed from  $2.0 \mu\text{m}$  to  $0.7 \mu\text{m}$ , which is as small as the focused Gaussian spot of a diffraction-limited microscopic objective of NA=0.7. We give a clear interpretation of the excitation spectrum of single atoms trapped in the BFORT, which is different from the Gaussian FORT spectrum. We then discuss the details of heating and scattering rate in the BFORT, and details of laser cooling the single atoms down to the Lamb-Dicke regime. We finally discuss how this approach can be used for quantum information processing.

## 2. BFORT: theoretical model

For a linearly polarized laser detuned far from the resonance frequency of  $^{87}\text{Rb}$ , the optical dipole potential  $U_{dip}(r, z)$  is calculated from [11],

$$U_{dip}(r, z) = \frac{\hbar\Gamma^2}{24I_s} \frac{I(r, z)}{\Delta}, \quad (1)$$

where  $\Gamma$  is the natural linewidth of  $^{87}\text{Rb}$ ,  $I_s$  is the saturation intensity,  $I(r, z)$  is the local intensity of laser,  $\Delta$  is the effective detuning given by  $\frac{1}{\Delta} = (\frac{1}{\Delta_{1/2}} + \frac{2}{\Delta_{3/2}})$ . The detuning  $\Delta_{1/2}$  ( $\Delta_{3/2}$ ) represents the difference between the laser frequency and the  $D_1$  ( $D_2$ ) transition frequency 795 nm (780 nm). For a red-detuned dipole potential ( $\Delta < 0$ ), the atoms are attracted to the intensity maximum with  $U_{rmax} < 0$ , and the trap depth  $\hat{U} = |U_{rmax}|$  is larger than atomic thermal energy  $k_B T$ . For a hollow blue-detuned dipole potential ( $\Delta > 0$ ), the dipole force repels atoms out of the field, and the potential minima correspond to the minima of the intensity. In this case, the potential depth  $U_{bmax}$  is determined by the height of the repulsive walls surrounding the center of the potential, which causes a dipole force that pushes the atoms to the center of the potential. So it may be possible to combine the red-detuned attractive dipole force and blue-detuned repulsive dipole force to form a stronger force for the atoms. This is easy to implement using a blue-detuned hollow beam of small size and high potential depth to ‘sharpen’ the side of the

radial trapping volume for the atoms. We choose to use the Laguerre-Gaussian laser beams, because they are the most robust and stable hollow beams.

Laguerre-Gaussian modes  $LG_p^l$  with azimuthal mode number  $l$  and radial mode number  $p$ , have been used to guide and trap cold atoms. For  $p = 0$ ,  $l \neq 0$ , the beams have a spiral phase structure where the phase is undefined on the optical axis [12]. A cross section of these beams looks like a doughnut, with a dark spot enclosed by a bright ring. The radial position of the maximum potential is related to  $l$  by  $r_{max} \propto \sqrt{l/2}$ . This indicates that a doughnut beam with  $l = 1$  will have the smallest dark spot. We will see that imposing  $LG_0^1$  can greatly squeeze the trapping dimension of single atoms in a Gaussian dipole trap. Throughout this paper we use the term doughnut beam to mean the  $LG_0^1$  beam. At the focus, the optical dipole potential  $U_b(r)$  of  $^{87}\text{Rb}$  atoms in a blue detuned doughnut beam can be written in terms of the maximum potential depth  $U_{bmax}$  as,

$$U_b(r) = eU_{bmax} \frac{2r^2}{w_{10}^2} \exp\left(-\frac{2r^2}{w_{10}^2}\right), \quad (2)$$

where  $U_{bmax} = \frac{\hbar\Gamma^2}{24I_s} \frac{I_{max}}{\Delta}$ , and  $I_{max}$  is the maximum intensity of a blue-detuned doughnut beam and given by  $I_{bmax} = \frac{2P}{e\pi w_{10}^2}$  at the focus,  $P$  is the total laser power,  $w_{10}$  is the waist size of the laser beam and  $e$  is the Euler's number  $e \approx 2.718$ .

Similarly, the optical potential  $U_r(r)$  for a red-detuned Gaussian beam has the form in terms of maximum potential depth  $\hat{U}$

$$U_r(r) = -\hat{U} \exp\left(-\frac{2r^2}{w_0^2}\right). \quad (3)$$

The total optical dipole potential is given by the superposition of these two optical dipole potentials with opposite signs,

$$U_{total}(r) = U_b(r) + U_r(r) = eU_{bmax} \frac{2r^2}{w_{10}^2} \exp\left(-\frac{2r^2}{w_{10}^2}\right) - \hat{U} \exp\left(-\frac{2r^2}{w_0^2}\right). \quad (4)$$

If the mean kinetic energy  $k_B T$  of a single atom is much smaller than the potential depth  $\hat{U}$ , the extension of a single atom is radially small compared to the beam waist. In this case, the optical potential Eq. (4) can be approximated by a harmonic oscillator as follows,

$$U_{total}(r) \approx -\hat{U} + \frac{2(eU_{bmax}w_0^2/w_{10}^2 + \hat{U})r^2}{w_0^2}. \quad (5)$$

The radial oscillation frequency  $\omega_r$  of a single atom trapped in the BFORT can be obtained from Eq. (5).  $\omega_{eff}$  can be expressed simply in terms of the gain factor defined by  $g = (eU_{bmax}w_0^2/\hat{U}w_{10}^2 + 1)^{1/2}$  and the oscillation frequency in the Gaussian FORT  $\omega_r = (4\hat{U}/mw_0^2)^{1/2}$ , that is:

$$\omega_{eff} = g\omega_r = \left(\frac{eU_{bmax}}{\hat{U}} \cdot \frac{w_0^2}{w_{10}^2} + 1\right)^{1/2} \left(4\hat{U}/mw_0^2\right)^{1/2}. \quad (6)$$

From Eq. (6), we find that for  $U_{bmax}/\hat{U} = 1$  and  $w_0^2 = w_{10}^2$ , we can achieve a trap steeper than the Gaussian trap and enhance the oscillation frequency by  $\sqrt{e+1} \approx 1.9$ . For  $U_{bmax}/\hat{U}=10$  and  $w_0^2 = w_{10}^2$ , we obtain enhancement of a factor of  $\sqrt{10 \times e + 1} \approx 5.3$ . So if the original waist of the focused Gaussian spot is  $1 \mu\text{m}$ , then the effective trapping waist for single atoms can be amazingly reduced to  $0.2 \mu\text{m}$ . This value is much smaller than the diffraction-limited spots  $1.22\lambda/\text{NA}$  of the objective with maximum numerical aperture  $\text{NA}=1$  and focused dipole laser wavelength  $\lambda = 830 \text{ nm}$ .

### 3. BFORT: experimental demonstration

Given the above theoretical analysis, we turn to our experimental details which tests this theoretical model. We used a microobjective with NA=0.38 placed outside the vacuum chamber to focus the dipole laser beams. Because of the aberration caused by the glass cell of the ultra high vacuum chamber, the focused system does not work in the diffraction-limited regime. A sketch of our setup is shown in Fig. 1.

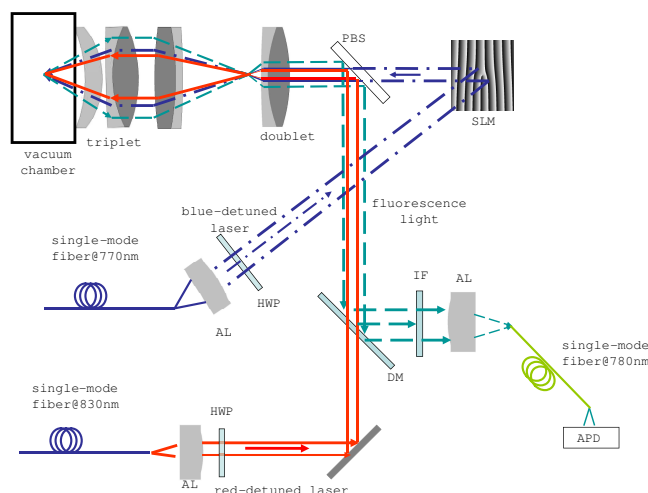


Fig. 1. Optical setup of the trapping (solid line) and imaging (dashed lines) systems. For details see the text.

The blue-detuned beam ( $\lambda = 770$  nm) from a tunable Ti:Sapphire laser, first passes through an acoustic optical modulator (AOM), and then is coupled into single mode polarization maintaining fiber and sent to the main optical table. On the main table, the blue detuned symmetric Gaussian laser beam is collimated by an aspherical lens (AL), then reflected by a Spatial Light Modulator (SLM, HOLOEYE HEO 1080P) with first-order diffraction efficiency of 40%. To generate the doughnut beam, a “fork” type phase hologram is displayed on the SLM. The calculation details of this phase hologram are in our recent work [13]. The SLM phase modulation creates a “charge-one” phase singularity in the beam, centered around the “fork” defect. A half-wave plate (HWP) rotates the light polarization axis to match the polarization required by the SLM. The doughnut beam is then expanded by two doublets and focused onto the magneto-optical trap (MOT) region by a triplet with NA=0.3 in image space.

The red-detuned dipole laser beam at 830 nm from a single mode laser is spatially filtered by a single mode-polarization-maintaining fiber. The fiber improves the beam quality of the diode laser beam significantly and provides the light that forms the radially symmetric trap for single atoms. The collimated red-detuned linearly polarized Gaussian beam is combined with the doughnut beam by a polarization beam splitter (PBS) so that both beams are confocal. The triplet is also used as an imaging system to collect the fluorescence (780 nm) of the atoms. The fluorescence is separated from the blue-detuned doughnut by a PBS and the red-detuned Gaussian beams by a dichroic mirror (DM). Then it passes through an interference filter (IF) used to block the stray dipole trap lights before entering the spatial filter of a single mode fiber. Finally, the fluorescence is detected by an avalanche photodiode (APD) assembled in a single photon counting module (SPCM, AQRH-14-FC).

We used a group of aberration free microscopic objectives with a magnification of 160x to

monitor the dipole traps in the vacuum. The images we obtained are shown in Fig. 2. The

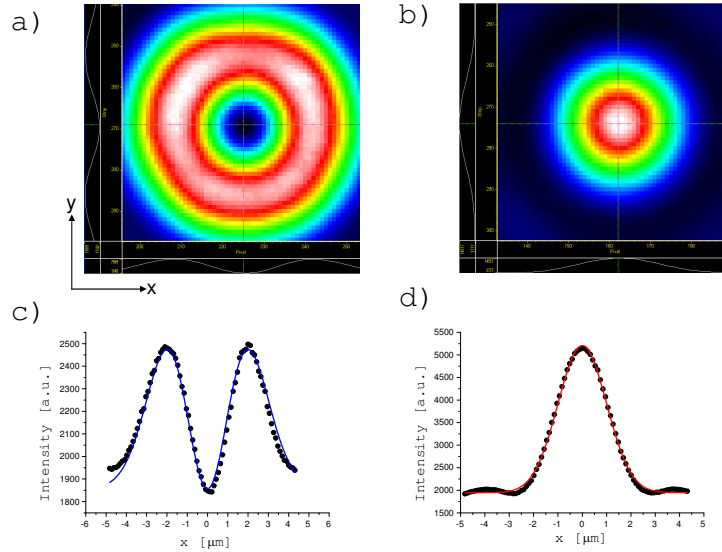


Fig. 2. (a) and (b) are the images of focused doughnut spot and radially symmetric Gaussian spot, taken with a linear CCD camera with 16-Bit resolution. (c) and (d) are the  $LG_0^1$  line profile fit and Gaussian line profile fit.

position of blue-detuned maximum intensity  $r_{max}$  is about  $1.62 \mu m$  as shown in the CCD image Fig. 2(a). The waist of the doughnut beam can be estimated from the relation  $r_{max} = \sqrt{l/2}w_{10}$  to be  $w_{10} \approx 2.29 \mu m$ . The waist of the doughnut beam can be also directly extracted from a line profile fit of  $LG_0^1$ , as shown in Fig. 2(c), and is about  $2.30 \pm 0.01 \mu m$ . We find the waist of Gaussian FORT to be about  $w_0 \approx 2.0 \mu m$  through measurement as shown in Fig. 2(b,d). With this waist, the red-detuned potential trap depth, proportional to the laser power, is  $\tilde{U} \sim 1.0$  mK for a 10 mW laser and the radial oscillation frequency is about  $\nu_r \sim 50$  kHz. This optical dipole trap allows us to trap single  $^{87}\text{Rb}$  atoms via a collisional blockade mechanism, which prevents two or more atoms from being trapped simultaneously due to optically assisted inelastic collisions [2]. We recently succeeded in trapping single  $^{87}\text{Rb}$  atoms in a red-detuned dipole trap and a blue-detuned optical bottle beam trap with 1 mK optical potential and proving the trapping property by observing the photon statistics of fluorescence with a Hanbury-Brown-Twiss setup [14]. The trapping experimental setup details are as in our previous work [13, 15].

The radial oscillation frequency  $\omega_{eff}$  of a single atom trapped in our BFORT was measured by parametric excitation of the oscillatory motion. Because of the repulsive potential for atoms from a blue-detuned doughnut beam, we first loaded a single atom into the red-detuned dipole trap with 1mK potential depth from the MOT while the doughnut beam was off. Once a single atom in the Gaussian trap was detected, we ramped up the doughnut beam over 10 ms. The imposed power of the doughnut beam was 42 mW which created a potential depth  $U_{bmax} = 3.9$  mK. From Eq. (6), we calculated that the oscillation frequency of a single atom trapped in 1 mK red-detuned potential would be enhanced by a factor of 3. Then the MOT beams were turned off and the rf power was sent to a high-voltage amplifier that drives an electro-optical amplitude modulator (EOM) to modulate the power of the Gaussian beam. After modulating red-detuned potential depth by 12% with one rf burst of 10000 cycles, we turned on the MOT beams to

detect the single atoms.

Surprisingly, given a constant amplitude modulation, we found that the intensity of resonance around 150 kHz was stronger than the resonance around 300 kHz. The resonance was so strong that it led to saturation, where all of the atoms in a large frequency range escaped so that we could not find the resonance peak. Therefore, to obtain a clear heating loss spectrum around 150 kHz, we scanned the modulation frequency from 200 to 450 kHz with 12% amplitude modulation and from 20 to 190 kHz with a reduced 1.9% amplitude modulation. The average survival probability is shown in Fig. 3(a), where we took about 100 measurements with one atom for each value of  $\omega$ . The clearly visible dips at  $\nu_r = 150$  kHz and  $\nu_r = 300$  kHz correspond to strong direct resonance and parametric resonance respectively. The full width at half maximum (FWHM) of the 300 kHz peak is about 30.0 kHz. In order to compare the BFORT and Gaussian FORT heating mechanisms, the spectrum of single atoms trapped in symmetric Gaussian FORT is shown in Fig. 3(b). The FWHM of the 100 kHz peak is about 15.6 kHz. For the heating loss spectrum in Gaussian FORT, the potential depth was modulated by 28% with one rf burst of 10000 cycles.

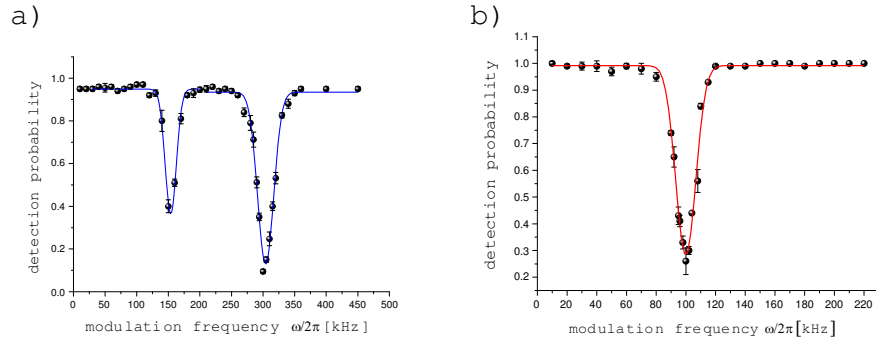


Fig. 3. Experimental spectrum of the vibrational frequencies of single atoms in the BFORT (a), and a Gaussian FORT (b). The filled circles depict the probability measurement of the single atoms in the traps after the modulation of the potential depth as a function of the modulation frequency. The solid curves are Gaussian fits to the experimental data.

We know that for a linear oscillator, parametric resonance occurs for modulation frequencies within a certain range  $\delta$  on either side of  $2\nu_{trap}$ . The width  $\delta$  is proportional to the modulation amplitude  $h_m$ , as  $\delta = h_m\nu_{trap}$ . The amplification coefficient of the resonance in this range is of the order of  $h_m$  [16]. So the FWHM of the parametric resonance are 14 kHz for the FORT and 18 kHz for the BFORT. The width of our experimentally measured Gaussian FORT resonance is close to the predicted value, but the width of one of our measured BFORT resonance is not. It is possible that parametric heating increases the energy of the single atoms and the BFORT deviates from the harmonic approximation. In that case, the motion of the single atoms can be modeled as an anharmonic oscillator which has a wider heating spectrum [16]. Besides the parametric resonance process occurs around  $2\nu_{trap}$ , parametric resonance also occurs around the frequencies  $\nu_{mod} = 2\nu_{trap}/n$ , where  $n > 1$  is integer, called ‘subharmonic’ resonance. But the width of resonance range decreases rapidly with increasing  $n$ , as the order of  $h_m^n$ . The amplification coefficient of resonance also decreases [16]. So it is reasonable that there is only one clear heating loss dip around the 100 kHz for FORT under the value of modulation degree.

Thus, the experimental results of the BFORT around 150 kHz do not match the characteristics of parametric resonance. The mechanism of the heating loss spectrum around 150 kHz is different from that of parametric resonance, though both result from modulating the trap depth.

The BFORT is different from the simple FORT since it is the combination of two different laser beams. Because of experimental imperfections, there will be a certain displacement between the optical axes of the Gaussian and doughnut beams. The equilibrium position of a single atom trapped in BFORT may depend on the ratio of the attractive potential to the repulsive one. When the Gaussian beam is modulated, the equilibrium position of a trapped single atom will shake accordingly. In the case of equilibrium position instability, shaking the potential at the trap oscillation frequency increases the oscillation amplitude and leads to heating [10, 16].

To prove this conjecture, we rewrite Eq. (4), adding the relative displacement degree  $r_d$ ,

$$U_{total}(r+r_d) = U_b(r) + U_r(r+r_d) = eU_{bmax} \frac{2r^2}{w_{10}^2} \exp\left(-\frac{2r^2}{w_{10}^2}\right) - \widehat{U} \exp\left(-\frac{2(r+r_d)^2}{w_0^2}\right) \quad (7)$$

In the harmonic oscillator approximation, the potential  $U_{total}(r+r_d)$  with a little displacement Eq. (7) can be rewritten as:

$$U_{total}(r+r_d) = U_b(r) + U_r(r+r_d) \approx -\widehat{U} + \frac{2}{w_0^2} (eU_{bmax}w_0^2/w_{10}^2 + \widehat{U}) \left( r + \frac{r_d\widehat{U}}{eU_{bmax}w_0^2/w_{10}^2 + \widehat{U}} \right)^2 \quad (8)$$

From Eq. (8), we see that the equilibrium position  $r_0 = r_d\widehat{U}/(eU_{bmax}w_0^2/w_{10}^2 + \widehat{U})$  does shake periodically when we modulate the trap depth  $\widehat{U}$ , so the atom trapped in the BFORT will undergo forced oscillating motion. When the modulation frequency is equal to the intrinsic trap frequency of the single atom, the energy of the atom will increase linearly over time [16].

The above theoretical analysis confirms that the dip in the heating loss spectrum at 150 kHz is a forced resonance, and the dip at 300 kHz is a parametric resonance. So the intrinsic oscillation of the single atoms in the BFORT is 150 kHz which agrees reasonably well with the theoretical expectation of  $\omega_{eff} = 2\pi * 150$  kHz. It is obvious that the doughnut beam enhances the oscillation frequency in Gaussian FORT by a factor of 3. Equivalently, the waist of the trapping region for a single atom is about 0.7  $\mu\text{m}$ , which is as small as diffraction-limited value of  $\text{NA}=0.7@830$  nm.

The relative displacement degree  $r_d$  can be estimated by using parametric and resonant heating mechanisms to calculate the corresponding heating rate which fits the heating loss spectrum as shown in Fig. 3(a). The parametric heating causes the average energy of single atoms to rise exponentially as,

$$\langle E(t) \rangle = \langle E(0) \rangle e^{\Gamma_\varepsilon t}, \quad (9)$$

where  $\Gamma_\varepsilon$  is the heating rate constant.  $\Gamma_\varepsilon$  can be determined using first-order time-dependent perturbation theory to calculate the average transition rates between quantum states of the trap and given by [10],

$$\Gamma_\varepsilon = \pi^2 v_{trap}^2 S_\varepsilon(2v_{trap}), \quad (10)$$

where  $S_\varepsilon(2v_{trap})$  is the one-sided power spectrum of the fractional intensity noise. Similarly, the forced oscillating motion will bring about linearly increasing in the average energy of single atoms. An energy-doubling time  $T_x$  can be defined as the time needed to increase the energy by the average energy at  $t=0: \langle \dot{E} \rangle / \langle E(0) \rangle \equiv 1/T_x$ .  $T_x$  is given by [10],

$$1/T_x = \pi^2 v_{trap}^2 \frac{S_x(v_{trap})}{\langle x^2 \rangle}, \quad (11)$$



where  $S_x(v_{\text{trap}})$  is the one-sided power spectrum of the position fluctuations in the BFORT center,  $\langle x^2 \rangle$  is the mean-square position of single atoms in the BFORT at  $t=0$  and is determined by the atomic mean energy. With the typical temperature  $80 \mu\text{K}$  of single  $^{87}\text{Rb}$  atoms in BFORT, the  $\langle x^2 \rangle$  is about  $0.09 \mu\text{m}$ . From the Eq. (8), the amplitude of  $S_x(v_{\text{trap}})$  is dependent not only on the modulation amplitude of the red-detuned laser intensity but also on the relative displacement degree  $r_d$ .

For  $v_{\text{trap}}=150 \text{ kHz}$ , the measured  $S_\varepsilon(2v_{\text{trap}})$  is about  $1.62 \times 10^{-3} \text{ frac}^2/\text{Hz}$  with 12% amplitude modulation and gives the parametric heating rate  $\Gamma_\varepsilon \approx 3.60 \times 10^8 \text{ s}^{-1}$  under the condition of exact resonance. In the experiments, the resonant heating rate with 12% amplitude modulation was larger than the parametric heating rate, but with the 1.9% amplitude modulation, the resonant heating rate was weaker than the latter. The  $r_d$  should make the resonant heating rate to satisfy the experimental observation. After optimization, we found that, when  $r_d \approx 0.15 \mu\text{m}$ , the 12% amplitude modulation and the 1.9% amplitude modulation made  $S1_x(v_{\text{trap}}) \approx 9.55 \times 10^{-5} \mu\text{m}^2/\text{Hz}$  and  $S2_x(v_{\text{trap}}) \approx 2.63 \times 10^{-6} \mu\text{m}^2/\text{Hz}$  respectively. For  $S1_x(v_{\text{trap}})$  and  $S2_x(v_{\text{trap}})$ , the calculated resonant heating rate are  $2.61 \times 10^9 \text{ s}^{-1}$  and  $7.21 \times 10^7 \text{ s}^{-1}$  respectively. We see that the  $r_d \approx 0.15 \mu\text{m}$  does match the experimental spectrum. We note that our auxiliary adjustment optics has a magnification of 160x and has about  $0.1 \pm 0.05 \mu\text{m}$  positioning accuracy.

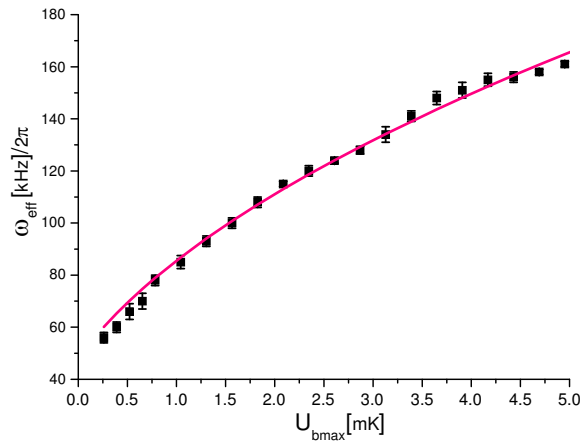


Fig. 4. The square root law of oscillation frequency enhancement. The filled squares are the experimental data obtained for a 1 mK red-detuned potential depth, and the line is from the theoretical model.

To show that, as described by Eq. (6), the frequency enhancement factor depends on the ratio of the blue-detuned potential depth to red detuned potential depth, we fixed the red-detuned potential depth at 1 mK and measured the oscillation frequency at different doughnut beam potential depths. Figure 4 shows our measurement data and the theoretically-calculated line. The data matches the theoretical calculation well. This finding allows us to adjust the radial oscillation frequency continuously by changing the blue-detuned potential depth. In summary, we have explained the characteristic heating loss spectrum and verified our theoretical model.

#### 4. Heating and scattering processes in the BFORT

In order to obtain further strong confinement of a single neutral atom, we utilized a small dark spot and higher blue-detuned potential to ‘sharpen’ the Gaussian beam. As is the case for the red-detuned dipole light, the absorptive part of the dipole interaction in blue-detuned light leads to residual photon scattering. This may lead to increased scattering heating and Raman spontaneous scattering rates, which decrease the lifetime of the hyperfine ground state. Now we discuss whether the added scattering limits the performance of the BFORT.

First, we measured the lifetime of the single atoms in the FORT and BFORT without the MOT beams. The study can help to understand how different heating mechanisms affect lifetime of the atoms in the trap. The data is shown in Fig. 5. From the measurement we know that the lifetime of single atoms trapped in the BFORT is about half that of those trapped in the Gaussian FORT.

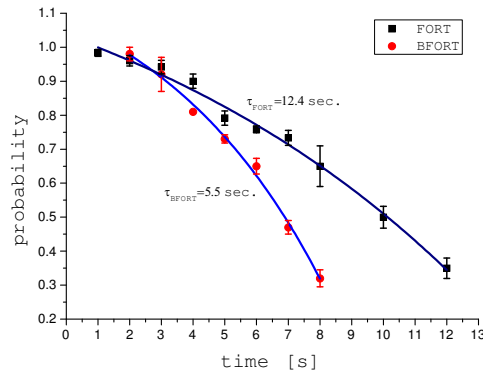


Fig. 5. The lifetime measurement of single atoms without the MOT cooling light.

There is a simple relation between the scattering rate  $\Gamma_{sc}$  and the dipole potential  $U_{dip}$  [17],

$$\Gamma_{sc} = \frac{\Gamma}{\hbar\Delta_{blue}} U_{dip}. \quad (12)$$

The mean scattering rate of the blue-detuned light  $\bar{\Gamma}_{sc}$  for single atoms at thermal equilibrium with temperature  $T_0$  trapped in the BFORT is [17]

$$\bar{\Gamma}_{sc} = \frac{\Gamma}{\hbar\Delta_{blue}} (U_0 + \frac{3}{2}k_B T_0), \quad (13)$$

where  $U_0 \approx 0$  accounts for the doughnut beam. This expression suggests that the mean rate of scattering blue-detuned light depends on the atoms’ temperature but not the blue detuned potential depth. The spontaneous scattering light of the doughnut beam can cause heating and increasing relaxation between hyperfine ground states of atoms. Both effects are determined by the mean scattering rate. For the heating rate, we have [17],

$$\dot{T}_{blue} = \frac{1}{2} T_{rec} \frac{\Gamma}{\hbar\Delta_{blue}} k_B T, \quad (14)$$

where the recoil temperature  $T_{rec} = \hbar^2 k_{blue}^2 / m$  is defined as the temperature associated with the kinetic energy gain by the emission of one blue-detuned photon. The equation shows that

heating causes the temperature to rise exponentially in a blue-detuned potential. Given the initial temperature  $T_0$  of atoms, we find the time-dependent temperature to be,

$$T_{blue}(t) = T_0 \exp\left(\frac{1}{2} \frac{T_{rec}\Gamma}{\hbar\Delta_{blue}} k_B t\right). \quad (15)$$

For our experimental parameter of  $\lambda_{blue} = 770$  nm, the time scale of the heating process is

$$T_{blue}(t) = T_0 e^{t/\tau}, \quad (16)$$

where the  $\tau \approx 17.3$  s. From this equation, we see that  $T_0$  will increase by a factor of e in about 17 seconds.

Unlike the exponential temperature of atoms scattering blue-detuned doughnut beam, the Gaussian FORT shows a constant rate [17]

$$\dot{T}_{red} = \frac{1}{3} T_{rec} \frac{\Gamma}{\hbar\Delta_{blue}} \hat{U}. \quad (17)$$

With parameters of wavelength 830 nm and  $\hat{U} = 1$  mK, we find the heating rate of the Gaussian FORT to  $\dot{T}_{red}(t) \approx 12$   $\mu$ K/s from the Eq. (17). For the initial temperature  $T_0 = 80$   $\mu$ K, scattering blue-scattering light cause the temperature to double in a shorter time than scattering blue-detuned light.

We can use a simple model to quantitatively calculate the lifetime of single atoms heated by scattering red and blue-detuned light in the BFORT. The energy distribution of the single atoms loaded from MOT and trapped in the dipole trap is thermal and follows the Boltzmann distribution [9]. The heating processes increase the temperature of the single atom as evidenced by repeating the experiment several times. Atoms then reach an energy higher than the trap potential depth  $\hat{U}$  and escape from the BFORT. The survival probability  $P_{surv}$  of the single atom remaining in the trap as the temperature rises is given by [9]:

$$P_{surv}(\xi) = 1 - (1 + \xi + 0.5\xi^2)e^{-\xi}, \quad (18)$$

where  $\xi = E/k_B T(t)$ ,  $E = |U_0| = 1$  mK is the maximum energy of single atoms trapped by the BFORT, and  $k_B T(t)$  is the time dependent mean energy of trapped atoms. For scattering blue-detuned light, the temperature is given by Eq. (16). So we can get the numerical probability of the single atoms remaining in the BFORT only heated by scattering blue-detuned light after time t with parameter  $T_0 = 80$   $\mu$ K, as shown by line 1 in Fig. 6. Similarly, we can get numerical result of the time dependent survival probability of single atoms heated by scattering red-detuned light, as shown by line 2 in Fig. 6. We find that the heating effect originating from scattering blue-detuned light is weaker than from scattering red-detuned light in the low temperature regime.

The second heating process that we must consider is the parametric heating caused by fluctuations in the trap depth. For a given  $S_\epsilon(2\nu_{trap})$ , the larger the oscillation frequency, the higher the heating rate and the shorter the lifetime of the single atoms trapped in the BFORT. To find the heating rate in the parametric resonance frequency range, we measured the  $S_\epsilon(2\nu_{trap})$  of the red-detuned laser at 100 kHz and 300 kHz, and obtained  $10^{-11.40}$  frac<sup>2</sup>/Hz and  $10^{-12.25}$  frac<sup>2</sup>/Hz respectively. Using these two values and the Eq. (10), we calculate the heating rate constant for a Gaussian FORT to be  $\Gamma_{FORT} \approx 0.098/s$  and the constant for BFORT to be  $\Gamma_{BFORT} \approx 0.135/s$ . We also measured  $S_\epsilon(2\nu_{trap})$  for blue-detuned laser and found out that it was one order of magnitude smaller than the  $S_\epsilon(2\nu_{trap})$  for the red-detuned laser, so it could be neglected. We then use Eq. (18) to calculate the time dependent survival probability of single atoms in the BFORT heated by a parametric resonance. The numerical results and experimental

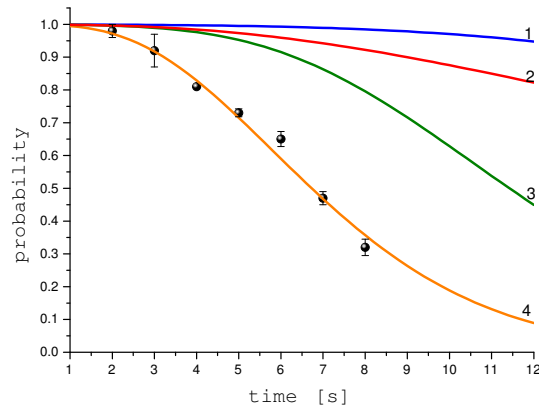


Fig. 6. Measured and theoretical survival probabilities for single atoms trapped in the BFORT. The filled circles are the measured probabilities of single atom remaining in the BFORT after time  $t$ . The lines 1 to 3 show the theoretical calculated survival probability for single atoms heated by scattering blue-detuned light (line 1), scattering red-detuned light (line 2), parametric resonance process (line 3). Line 4 is survival probability when all three effects are considered.

data are shown in Fig. 6. From the comparison in Fig. 6, we can see that the parametric heating process has the greatest influence on the lifetime of atoms trapped in the BFORT.

Furthermore, the equilibrium position of single atoms for an imperfect beam alignment depends on the intensity of both beams, so the relative intensity fluctuations will heat the single atoms too. For the relative displacement degree  $r_d \approx 0.15 \mu\text{m}$ , we measured the  $S_x(v_{\text{trap}})$  of the red-detuned laser at 150 kHz and obtained  $1.0 \times 10^{-15.0} \mu\text{m}^2/\text{Hz}$ . By using the Eq. (11), the heating rate of single atoms in BFORT dominated by the forced oscillating motion is about 0.027/s, so this heating effect can be neglected.

Finally, we sum up the three heating processes to obtain the total heating rate of the single atoms in the BFORT. The total heating rate then can be used to calculate single atoms' lifetime, which matches well with the lifetime measurement, as shown by line 4 in Fig. 6.

The calculated results clearly confirm our model. We note that in any harmonic trap used for trapping neutral atoms, enhancement of the trapping frequency will always give rise to a higher parametric heating rate. Thus, the shorter lifetime of single atoms in the BFORT does not indicate poor performance but successful oscillation frequency enhancement. However, in order to enhance the lifetime of a single atom strongly trapped in any dipole trap, trap depth stabilization in the high frequency range or further reduced temperature of single atoms need to be considered.

Besides elastic Rayleigh scattering, there is some percentage of scattering events, called spontaneous Raman scattering, which are quasi-elastic and change the atomic ground state [18]. The total scattering rate given by Eq. (13) is the sum of the Rayleigh and Raman scattering rates. The exact Raman scattering rate can be determined by the ratio of the Raman scattering rate  $\Gamma_{\text{Raman}}$  to the Rayleigh scattering rate  $\Gamma_{\text{Rayleigh}}$ . For the 770 nm doughnut beams wavelength, the ratio of mean Raman scattering rate to the mean total scattering rate  $\bar{\Gamma}_{sc}$  is calculated by the Kramers-Heisenberg formula [18] and is about 0.1. So mean rate of Raman scattering blue-

detuned light is proportional to the temperature of the single atoms and given by,

$$\Gamma_{Raman} \approx 0.10\bar{\Gamma}_{sc} \approx 0.10 \frac{3}{2} \frac{\Gamma}{\hbar\Delta_{blue}} k_B T_0 \approx 4.5 \text{ photons/s} \quad (19)$$

for a typical  $T_0 = 80 \mu\text{K}$ . The rate of Raman scattering red-detuned light is proportional to the trap depth and is about 1.3 photons/s for a 830 nm wavelength and 1 mK trap depth. It seems likely that the blue-detuned light contributes most to the Raman scattering rate. But unlike the Raman scattering rate in the red-detuned Gaussian FORT, the rate of scattering blue-detuned light can be nearly suppressed by further cooling the atoms down to the ground state.

## 5. Laser cooling of single atoms trapped in the BFORT

Now we turn to the field of laser cooling of single atoms trapped in the Gaussian FORT and BFORT. Using an optimized laser-cooling sequence Tuchendler et al. [9] were able to significantly reduce the temperature of the atoms from  $155 \mu\text{K}$  to  $31 \mu\text{K}$  for a trap depth of 2.5 mK. In our experiment, we cooled single atoms by implementing the normal sub-Doppler cooling process, which was accomplished by increasing the detunings and decreasing the intensity of the MOT light. Then we employed the release and recapture method to determine the mean energy of the single atoms in the FORT [9].

To obtain the coldest single atoms, we carried out the optimized laser cooling sequence as follows. Initially, we loaded a single atom into the Gaussian FORT with trap depth 0.5 mK. The atomic resonance frequency is light shifted by about  $-2\Gamma$  with respect to the free space case. Subsequently, the cooling laser was linearly detuned from  $-6\Gamma$  to  $-13\Gamma$  in 10ms, during this time the intensities of the cooling lasers were reduced to 20% of their initial intensity. After this cooling period, the cooling lasers and repump laser were switched off. Then we implemented a release and recapture experiment. Figure 7(a) shows the release and recapture experimental results after the laser cooling, together with the best-fit simulation results. This corresponds to a temperature of  $13 \pm 1 \mu\text{K}$  for a Gaussian FORT with oscillation frequency  $\omega_r/2\pi \sim 35 \text{ kHz}$ . So the mean number of radial oscillator is about  $\bar{n} = k_B T / \hbar \omega_r \sim 7.2$  and gives the relation  $\eta^2(2\bar{n} + 1) = 1.2$ . Obviously, the single atoms trapped in a shallower Gaussian FORT can be cooled to lower temperature, but it can not be brought into the Lamb-Dicke regime because of the relatively small oscillation frequency which gives related large  $\bar{n}$ .

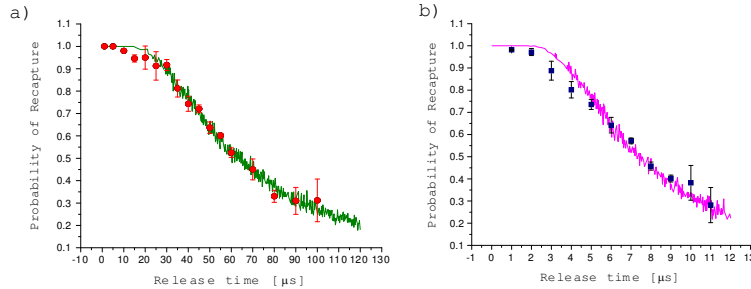


Fig. 7. Measurement of the temperature of the atoms in Gaussian FORT and BFORT using the release and recapture method. (a) and (b) are the temperature of the single atoms in Gaussian FORT and BFORT respectively. Each data point is the accumulation of 200 sequences. Superimposed on this data is a fit by the Monte Carlo simulation of the release and recapture method, which is the average of 500 trajectories for each release time. The temperatures of (a) and (b) are  $13 \pm 1 \mu\text{K}$  and  $15 \pm 1 \mu\text{K}$  respectively.

BFORT can enhance the oscillation frequency and hardly increases the light shift of atoms. This may help to bring the single atoms into the Lamb-Dicke regime through laser cooling single atoms in BFORT. After the above cooling process, we ramped up the doughnut beam over 10 ms and then implemented the above cooling process again. The LG potential height was about 3.9 mK and the oscillation frequency was about  $\omega_{eff}/2\pi \sim 145$  kHz. Figure 7(b) shows the release and recapture experimental results after the laser cooling in the BFORT, together with the best-fit simulation results. This corresponds to a temperature of  $15 \pm 1$   $\mu$ K for a BFORT with oscillation frequency 145 kHz. So the mean quantum number of the radial oscillator is about  $\bar{n} = k_B T / \hbar \omega_{eff} \sim 1.65$  and gives the relation  $\eta_{BFORT}^2 (2\bar{n} + 1) = 0.11$ . This indicates that the quantum number of the atomic radial oscillator state in BFORT can be efficiently reduced by laser cooling and eventually satisfies the Lamb-Dicke criterion. This is a good starting point for implementing a proposed protocol to entangle two trapped atoms through the emission of a single photon by one of the atoms [19] or implementing Raman sideband cooling [20] or EIT cooling [21] to further cool the single atoms down to their ground state.

Furthermore, we can optimize our scheme by using diffraction-limited optics as in [8]. The diffraction-limited waist of NA=0.5 at  $\lambda = 830$  nm is about 1.01  $\mu$ m. As proved by Eq. (6), in the case where  $w_0^2/w_{10}^2 = 1$  the oscillation frequency of a single atom trapped in 0.5 mK red-detuned potential can be raised to 328 kHz by imposing a 4 mK blue-detuned potential, the corresponding ground state temperature would then exceed 7  $\mu$ K, a temperature that can be approached by the normal sub-Doppler cooling process. The corresponding root mean square spread of atoms is  $\Delta x \approx 13$  nm  $\approx \lambda/60$  for  $\lambda = 780$  nm, and effective trapping region is close to what could be achieved by an optical lattice. To our knowledge, this is hardly obtained by using a normal high numerical aperture objective to focus a simple red-detuned Gaussian beam and form a shallow FORT. Our scheme is a good candidate for implementing quantum logic gates by using coherent dipole-dipole interactions between two trapped  $^{87}\text{Rb}$  atoms [22]. The two trapped atoms can be loaded efficiently into a red-detuned optical dipole trap by dynamically reshaping the trap with a spatial light modulator [23]. Our scheme is also suitable for compressing the radial direction of a 1-D optical lattice, in which a single atom has been cooled down to the ground state by microwave radiation [24], and results in 3-D strong confinement of single atoms.

## 6. Conclusion

In summary, we have proposed and experimentally demonstrated a tunable steep BFORT for single atoms. We have found that applying a blue-detuned doughnut beam increases the oscillation frequency of a single atom in a Gaussian FORT. The frequency enhancement is proportional to the square root of the ratio of blue-detuned potential depth to red detuned potential depth. The BFORT is an excellent scheme for enhancing the oscillation frequency of the single atom in any existing system, with the goal of further cooling to the ground state. Because the scattering rate of the same blue-detuned light mainly depends on the temperature of the atoms, it is reduced when the atoms are close to the ground state. We finally set the single atoms to be in the Lamb-Dicke regime by normal sub-Doppler laser cooling. Our work is in progress towards cooling atoms closer to the ground state for quantum information processing applications.

## Acknowledgments

The authors would like to thank Jean-Patrick Connerade and Sara Campbell for carefully reading the manuscript. This work was supported by the National Basic Research Program of China under Grant No.2012CB922101, by the National Natural Science Foundation of China under Grant Nos.11104320 and 11104321, and also by funds from the Chinese Academy of Sciences.